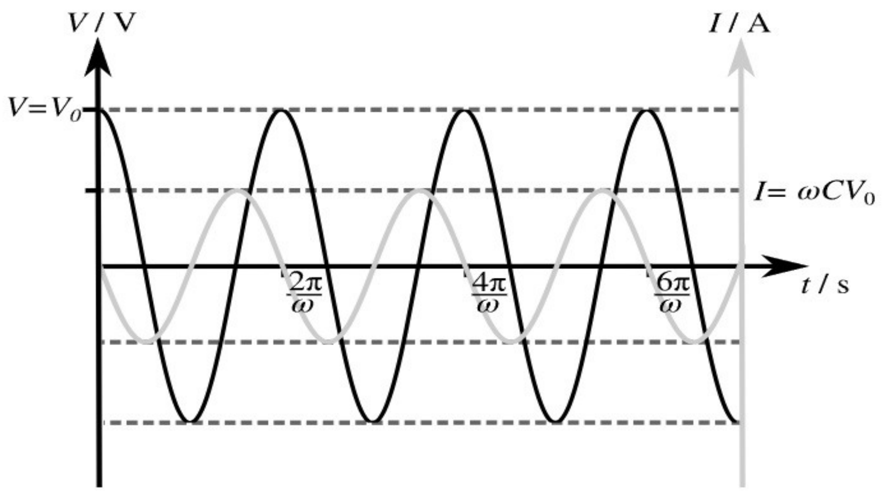
DC AND AC BEHAVIOUR OF CAPACITORS

When a voltage is applied to an un-charged capacitor a large current flows; it is as if the capacitor has no resistance.

But a charge quickly builds up on the plates until the voltage across the plates equals the supply voltage, then charging stops. Thus, when a capacitor is connected to a DC supply, there is a brief surge of current, but then the DC current is blocked.

When the capacitor is connected to an AC supply the voltage is constantly changing, so a stable situation is never reached.



Current is created as the charge on the capacitor changes. When the voltage is at its maximum it is not changing, so the there is no current.

As the voltage drops, charge flows out of the capacitor, so there is a negative current. This is greatest where the voltage is changing most rapidly as it crosses zero. Then the voltage change slows down, comes to a rest and start to go up again. This causes the current to reduce, pass through zero and start to go positive. This cycle repeats with the zero current corresponding to maximum voltage and vice-versa.

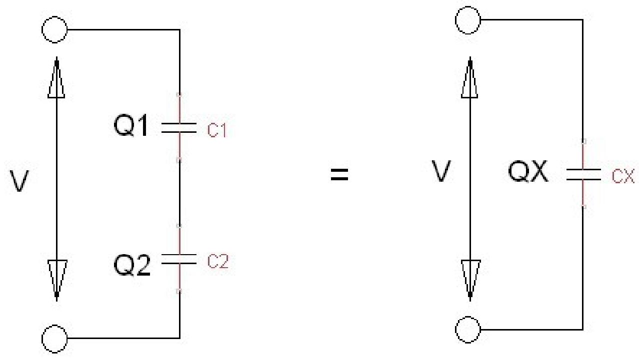
There is a 'phase shift' between the voltage and current. The current curve is ¼ cycle ahead of the voltage curve. The rule is: “In a capacitor, Current leads Voltage”

The 'resistance' of a capacitor depends on the frequency. We call this quantity its 'Impedance', with the symbol Z. It has the formula:

$$Z = \frac{1}{2\pi f C}$$

The higher the frequency, the smaller Z is, i.e. capacitors have less 'resistance' to high frequencies than low frequencies.

CAPACITORS IN SERIES

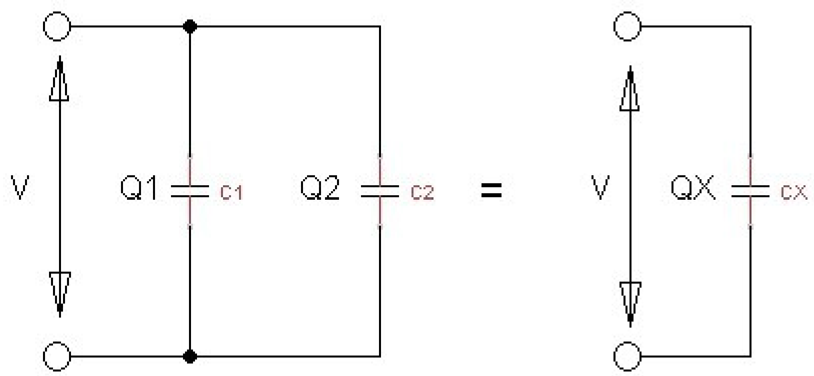


Consider 2 capacitors C1 and C2 connected in series, equivalent to a single capacitor CX. The formula is:

$$\frac{1}{CX} = \frac{1}{C1} + \frac{1}{C2}$$

Note that formula for capacitors in series is similar to the formula for resistors in parallel.

CAPACITORS IN PARALLEL



These capacitors are connected in parallel so they have the same voltage applied to each. The equivalent single capacitor CX is given by the formula:

$$CX = C1 + C2$$

Note that the formula for capacitors in parallel is similar to the formula for resistors in series.

REFERENCE SHEET: PROOFS OF CAPACITOR FORMULAS

Impedance:

Let the maximum voltage and current be V and I respectivly
Let the instantanious voltage and current at time t be v and i respectivly
Let the frequency be f Hz. Then the angular frequency $\omega = 2\pi f$ radians/sec
(1) for a sine wave $v = V\sin(\omega t)$
(2) i = rate of change of charge $= \frac{dQ}{dt}$
(3) by definition $Q = CV$ therefore $i = C\frac{dv}{dt}$
(4) from (2) and (3) $i = C\,V\frac{d(\sin(\omega t))}{dt} = C\,V\,\omega\cos(t)$
(5) the maximum value of $\cos(t)$ is 1 so $I = C\,V\,\omega$
(6) by definition, $Z = \frac{V}{I}$
from (5) and (6) $Z = \frac{V}{C\,V\,\omega} = \frac{1}{\omega C} = \frac{1}{2\pi f\,C}$

In the following, let the capacitors be $C1$ and $C2$, and the equivalent total capacitance be Ct
Let the current, voltage and charge for capacitor Cx be Ix, Vx and Qx
and, by definition, $Q = V.C$

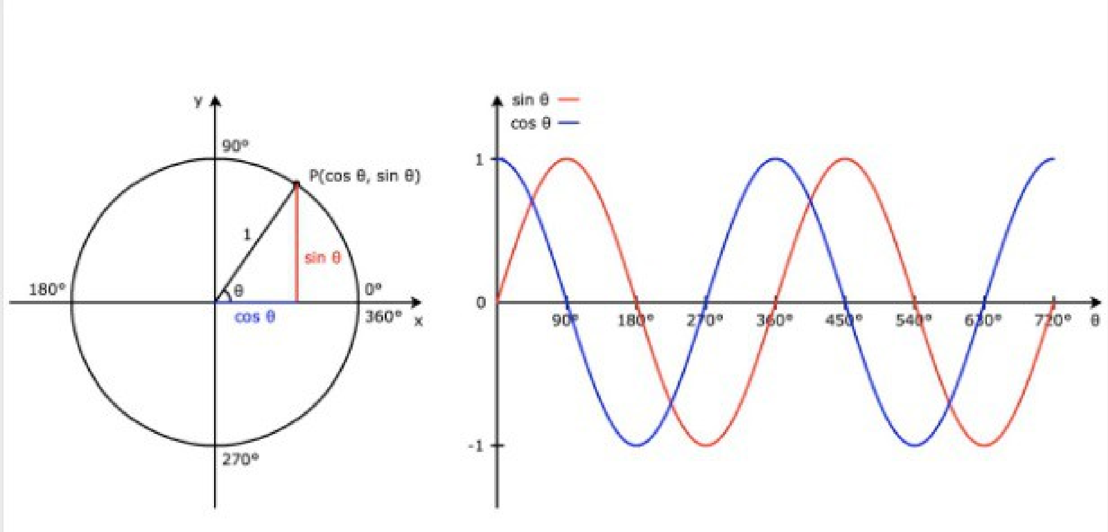
Capacitors in series.:

The same current flows through all the capacitors
(1) therefore, in any given time interval $Qt = Q1 = Q2 = Q$
(2) $Vt = \frac{Q}{Ct}, V1 = \frac{Q}{C1}, V2 = \frac{Q}{C2}$
(3) we require that $Vt = V1 + V2$
(4) from (2) and (3) $\frac{Q}{Ct} = \frac{Q}{C1} + \frac{Q}{C2}$ or $\frac{1}{Ct} = \frac{1}{C1} + \frac{1}{C2}$

Capacitors in parallel

The voltage across all the capacitors is the same
(1) $Vt = V1 = V2 = V$
(2) we require $It = I1 + I2$
(3) so $Qt = Q1 + Q2$
so $V.Ct = (V.C1) + (V.C2)$ or $Ct = C1 + C2$

REFERENCE SHEET: WAVE THEORY



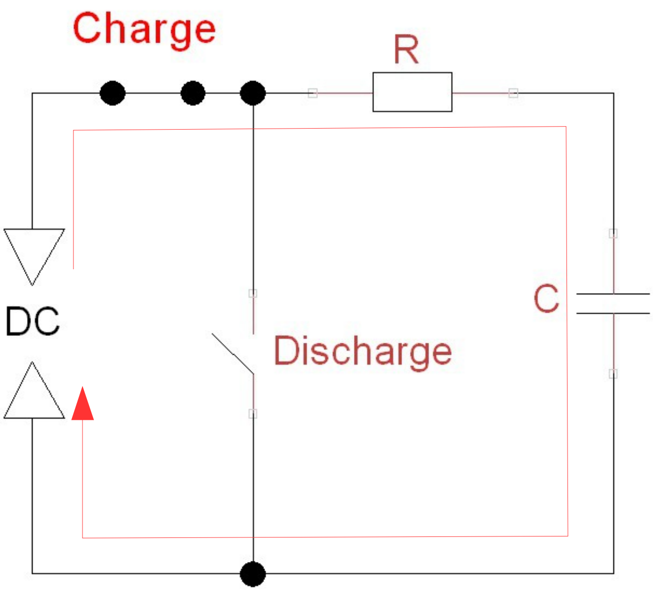
Given a circle or unit radius, we can imagine a radial line rotating about the centre. The distance from the end of the line to the horizontal axis is called the Sine of the angle (written as Sin, but pronounces as sine). Similarly the distance from the vertical axis is called the Cosine (Cos) of the angle. If we plot these values as the line rotates we get sin and cos waves. The 90° difference between the curves is called the Phase Angle. We could measure the distance to any sloping line through the centre of the circle and get a curve shifted by the corresponding phase angle.

RADIANS

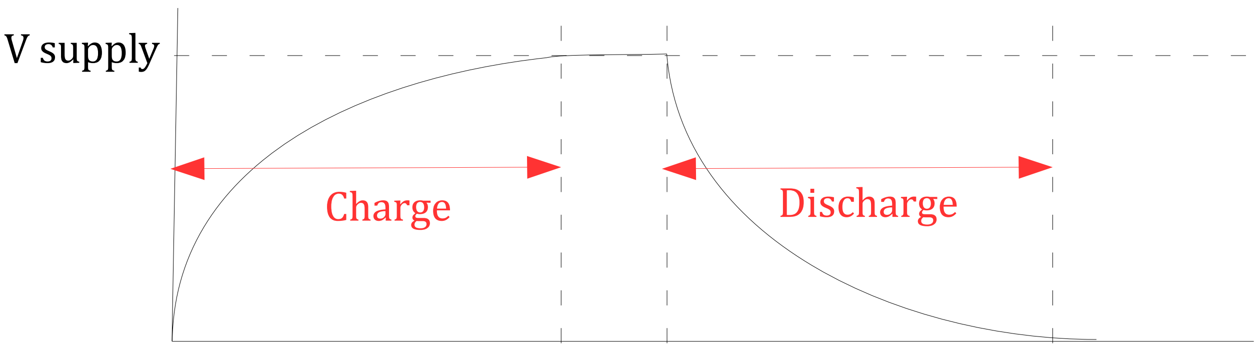
In the above diagram the angles are measured in degrees. The complete circle is divided into 360 degrees.

Calculations for waves are easier if we adopt a different measure of angle, called Radians. The complete circle is divided into 2π radians, i.e. 360° = 6.283 radians (to 3 decimal places). 180° = π radians. 90° = $\pi/2$ radians.

CHARGING A CAPACITOR THROUGH A RESISTOR



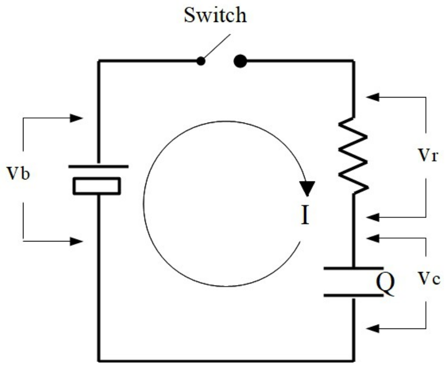
In this circuit, closing the switch labelled 'Charge' allows current to flow through the resistor to charge the capacitor. Initially, with no charge on the capacitor, the full supply voltage is present across the resistor, and the current = V/R. But as the voltage on the capacitor increases the voltage across the resistor decreases, and the current reduces, until, after a long time, the voltage on the capacitor is equal to the supply voltage and no current flows.



If the 'Discharge' switch is closed, the capacitor discharges through the resistor, in a similar way.

The value of C x R is called the 'time constant'. It always takes CR seconds for the voltage on the capacitor to reach, or fall by, 63% of the supply voltage. This is a very useful fact when designing timing or oscillating circuits.

REFERENCE SHEET: R-C TIME CONSTANT



We will adopt the usual convention that current flowing in the direction shown is the positive direction. Voltage across the battery (current flowing from negative to positive) will be considered to be positive, so the voltage drops across the resistor and capacitor (current flowing from positive to negative) is negative.

To analyse this circuit we use Kirchoff's second law (Voltage Rule): The sum of the voltage drops ΔV_i, across any circuit elements that form a closed circuit is zero:

$$\sum_{i=1}^{i=n} \Delta V_i = 0$$

INITIAL STATE:

At t = 0, the switch is closed. The capacitor initially is uncharged and acts like a short circuit.

$$At\ t=0,\ \ Q=0,\ \ V_c=0,\ \ I=\frac{V_b}{R}$$

FINAL STATE:

After a very long time has passed the voltage difference between the capacitor plates equals the battery voltage. Since there is no current between points of equal voltage, the current will be zero.

$$As\ t\to\infty,\ \ Q\to CV_b,\ \ V_c\to V_b,\ \ I\to 0$$

VOLTAGE AND CHARGE AS A FUNCTION OF TIME:

As the charge builds up on the capacitor plates, the current drops from the initial value to zero. Using the voltage rule, and observing the convention for the sign of the voltages, at time t we have:

$$V_b - V_r - V_c = 0 \ \text{ so } \ V_b - I_{tr} - \frac{Q_t}{C} = 0$$

Current is the flow of charge, i.e. the rate of change of charge with time. So the current is related to the charge on the capacitor by:

$$I = \frac{+dQ_t}{dt}$$

$$V_b - \frac{dQ_t}{dt}R - \frac{Q_t}{C} = 0 \ \text{ so } \ \frac{dQ_t}{dt}R = V_b - \frac{Q_t}{C} \ \text{ so } \ \frac{dQ_t}{dt} = \frac{1}{R}\left(V_b - \frac{Q_t}{C}\right)$$

This equation can be solved by the method of separation of variables. First put terms involving time (dt) on the right and the terms involving charge on the left:

$$\frac{dQ_t}{\left(V_b - \frac{Q_t}{C}\right)} = \frac{1}{R}dt$$

Re-arranging the terms on the left hand side to facilitate integration...

$$\frac{CdQ_t}{CV_b - Q_t} = \frac{1}{R}dt \ \text{ so } \ \frac{dQ_t}{CV_b - Q_t} = \frac{1}{RC}dt \ \text{ so } \ \frac{dQ_t}{Q_t - CV_b} = -\frac{1}{RC}dt$$

The value RC is called the Time Constant of the circuit, and given the symbol τ. With C in Farads and R in Ohms, τ is in seconds.

REFERENCE SHEET: R-C TIME CONSTANT (continued)

CV_b is the maximum value of Q as $t \to \infty$ which we will call Q_{max}

Integrating both sides of the above equation we get:

$$\int_0^{Q_t} \frac{dQ_t}{Q_t - Q_{max}} = -\frac{1}{\tau} \int_0^t dt$$

For the LHS, this type of integral is solved thus: $\int \frac{c}{ax+b} dx = \frac{c}{a} \ln(|ax+b|) + K$

Since $Q_{max} > Q_t$ then $|Q_t - Q_{max}| = Q_{max} - Q_t$ So we can evaluate the integrals thus:

$$\int_0^{Q_t} \frac{dQ_t}{Q_t - Q_{max}} = \ln(Q_{max} - Q_t) - \ln(Q_{max} - 0) = \ln\left(\frac{Q_{max} - Q_t}{Q_{max}}\right) = \ln\left(1 - \frac{Q_t}{Q_{max}}\right)$$

From initial conditions we can see that the arbitrary constant K = 0

For the RHS, this type of integral is solved thus: $\int dx = x$

$$-\frac{1}{\tau} \int_0^t dt = -\frac{1}{\tau}(t-0) = -\frac{t}{\tau} \quad \text{so} \quad \ln\left(1 - \frac{Q_t}{Q_{max}}\right) = -\frac{t}{\tau} \quad \text{so} \quad t = -\tau \ln\left(1 - \frac{Q_t}{Q_{max}}\right)$$

Noting that: $\frac{Q_t}{Q_{max}} = \frac{V_t}{V_b}$

we find that the time to charge the capacitor to a given voltage is:

$$t = -\tau \ln\left(1 - \frac{V_t}{V_b}\right) \quad \text{or} \quad -\frac{t}{\tau} = \ln\left(1 - \frac{V_t}{V_b}\right)$$

If $\ln(x) = y$ then $e^y = x$

We have $x = 1 - \frac{V_t}{V_b}$ and $y = -\frac{t}{\tau}$

so $e^{-\left(\frac{t}{\tau}\right)} = 1 - \frac{V_t}{V_b}$ so $\frac{V_t}{V_b} = 1 - e^{-\left(\frac{t}{\tau}\right)}$ or $V_t = V_b\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right)$

Setting $t = \tau$ we have $V_t = V_b(1 - e^{-1})$ or $V_t = 0.632 V_b$

It takes CR seconds for the voltage on the capacitor to reach 63% of the supply voltage.

POWER IN AC CIRCUITS

We know that the power in a DC circuit is simply given by V x A. But in an AC circuit the voltage and current are constantly changing, so the power output is also constantly changing. In devices such as heaters, light bulbs, motors etc., what matters is the average (or 'mean') power output.

We can compare the mean power output of an AC circuit to the power output of a DC circuit of the same power:

For DC: $P \alpha V^2$ (α means 'is proportional to')
For AC: $P \alpha \, mean(v^2)$

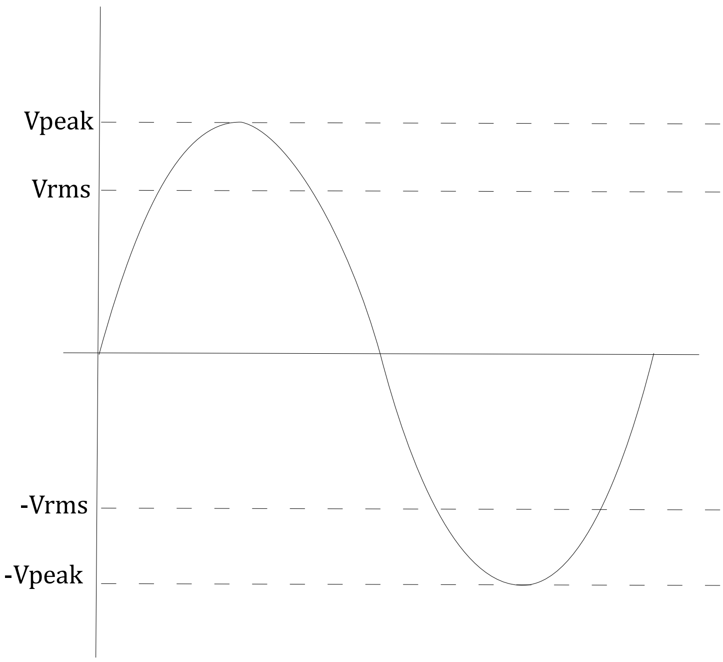
so $V^2 \alpha \, mean(v^2)$

or $V \alpha \sqrt{mean(v^2)}$

The value of $\sqrt{mean(v^2)}$ is called the root-mean-square, or RMS Voltage.

The ratio of Vpeak to Vrms depends on the waveform. For a sine wave

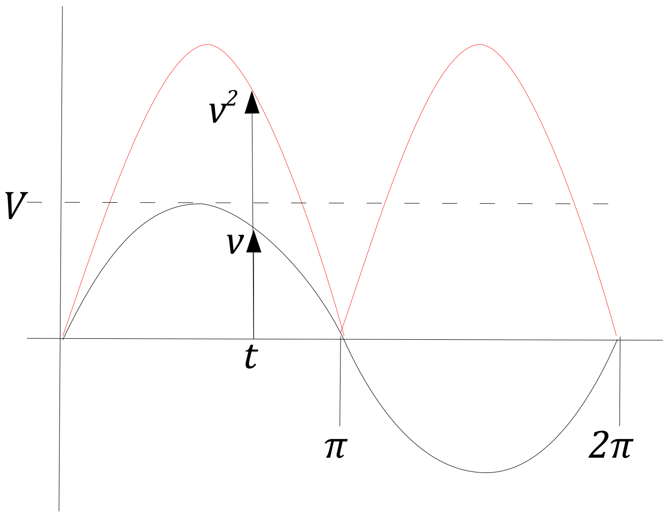
$$V_{RMS} = \frac{V_{PEAK}}{\sqrt{2}} \quad \text{or} \quad V_{PEAK} = V_{RMS} \times \sqrt{2} \qquad (\sqrt{2} = 1.414)$$



For UK mains electricity, the RMS voltage is 230V, so the peak voltage is 230 x 1.414 = 325V.

REFERENCE SHEET: RMS VOLTAGE

To show that the Vpeak = 1.414 x Vrms for a sine wave, consider a 1Hz sine wave (i.e. 2π radians/sec):



The voltage v at time t is given by $v(t)=V\sin(t)$

Squaring: $v^2(t)=V^2\sin^2(t)$

it can be shown that $\sin^2(t)=\frac{1-\cos(2t)}{2}$

so $v^2(t)=V^2\frac{1-\cos(2t)}{2}=\frac{V^2}{2}-\frac{V^2\cos(2t)}{2}$

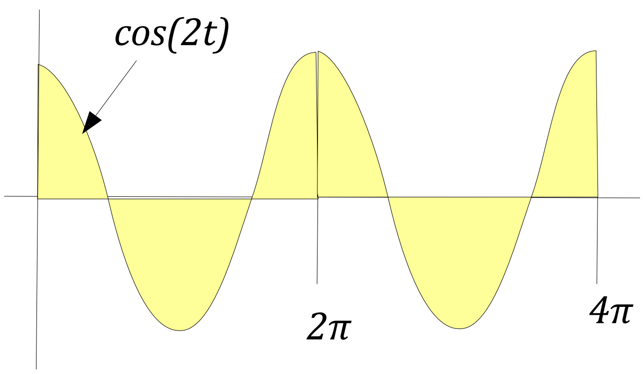
We want to find the mean between t=0 and t=2π.

We note that $mean(a+b)=mean(a)+mean(b)$

so $mean(v^2(t))=mean(\frac{V^2}{2})-mean(\frac{V^2\cos(2t)}{2})$

$\frac{V^2}{2}$ is a constant so it's mean is also $\frac{V^2}{2}$

The mean of cos() over any number of complete cycles is zero, which can easily be seen from this diagram, because the area above zero is equal to the area below zero. In our case, the mean of cos() from 0 to 4π = 0



Threfore $mean(v^2(t))=\frac{V^2}{2}$

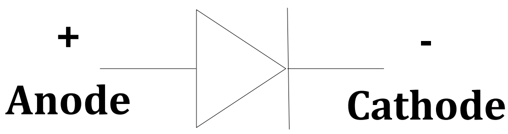
Taking the square-root: $\sqrt{mean(v^2(t))}=\sqrt{\frac{V^2}{2}}=\frac{V}{\sqrt{2}}$

<u>DIODES</u>

A Diode is a device that allows current to flow in one direction only. You can think of a diode as a one-way valve.

It gets its name from the days when diodes were a type of vacuum tube, and they were named after the number of connections, Diode = 2 connections, Triode – 3 connections, etc.

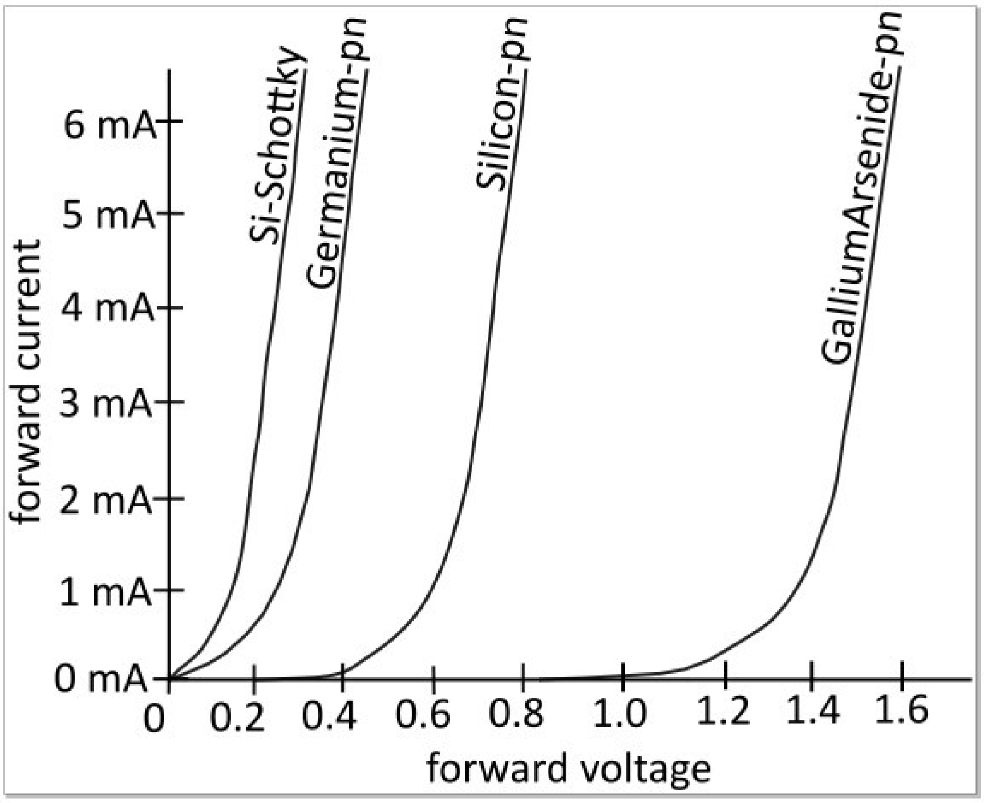
These days diodes are 'semiconductor' or 'solid state' devices.



The symbol for a diode indicates the direction of conventional current flow from positive to negative. This is called 'forward bias'. If the polarity is reversed (reverse bias) almost no current flows.

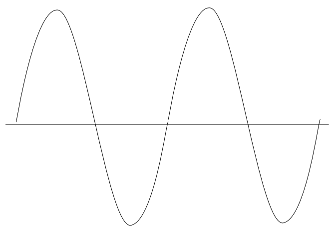
Small diodes look similar to resistors, but have no colour code. A single band (any colour) indicates the cathode terminal.

Unlike a resistor, the current is not proportional to the voltage. Rather than having a constant resistance, diodes have an almost constant voltage drop across them, called the 'Forward Voltage'. For example, a silicon diode does not conduct below 0.4 Volts and as the voltage increases only a little, the current rises rapidly. At about 0.8 volts the maximum current capacity of the diode is reached. This is called the 'Threshold Voltage'.

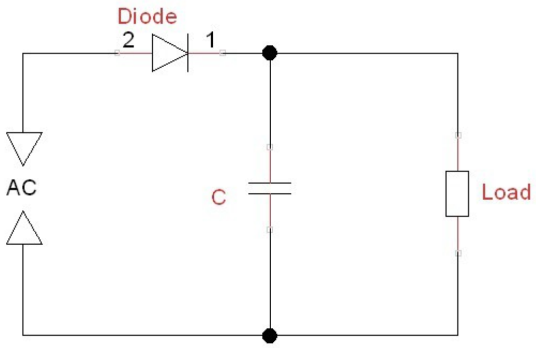


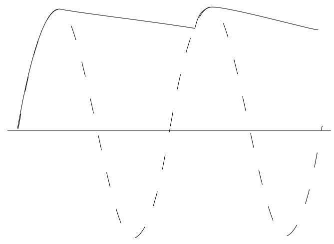
USES OF DIODES

Here is a typical use for a diode.



Supply voltage

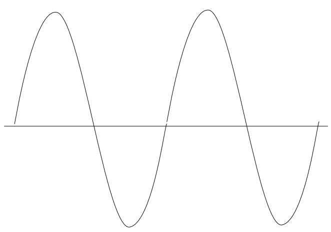


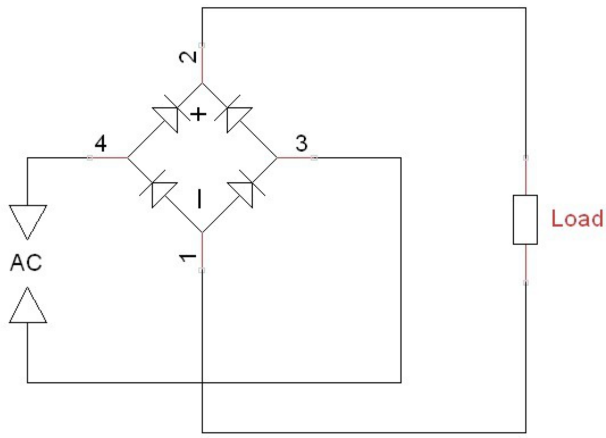


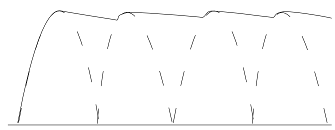
Voltage across load

When the supply voltage is greater than the voltage on the capacitor, current flows through the diode to charge the capacitor, but when the supply voltage drops below the capacitor voltage, the capacitor cannot discharge back through the diode. Providing the load does not draw too much current, the voltage on the load is kept fairly constant, near the peak voltage of the supply.

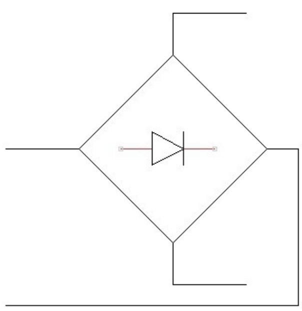
In the above, the capacitor is charged once per cycle. We can get a steadier voltage on the load by making it charge twice per cycle like this...







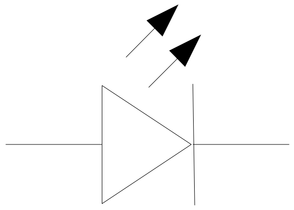
This arrangement of 4 diodes is called a 'bridge rectifier' and is so common that it is often given this simplified symbol:



LIGHT EMITTING DIODES

A Light Emitting Diode is a diode in which on of the layers of semiconductor material emits a photon when an electron passes through it.

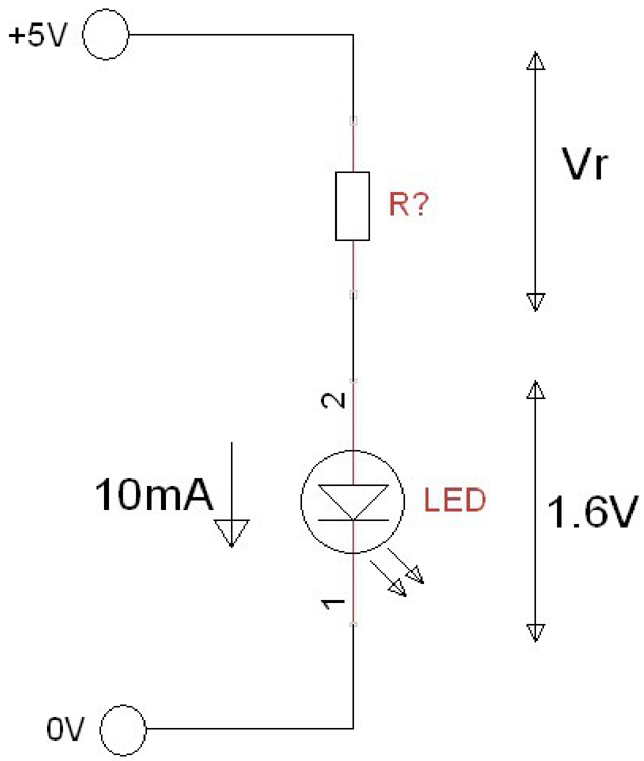
The extra energy required to release the photons means that the forward voltage of LEDs is increased to about 1.6 Volts.



The symbol for an LED has 2 arrows indicating the light emission.

CURRENT LIMITING RESISTOR

Diodes, including LEDs, have almost no resistance above the threshold voltage, so if an LED is connected straight across the power supply a very large current would flow, which would immediately burn out the device. It is always necessary to use other components to limit the current to within the allowed range for the LED. A simple resistor is usually used.



Here the LED has a threshold voltage of 1.6 volts and requires a current of 10mA. Calculate the value of R.

(Hint: first calculate Vr)

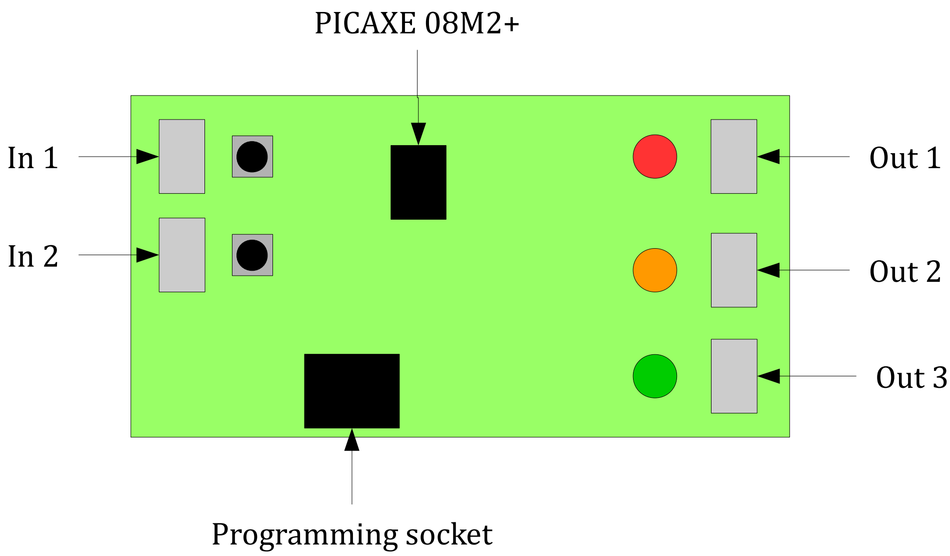
MICROPROCESSORS

Microprocessors are 'a computer on a chip'.

Because they are programmable, they allow you to create devices that would need a large and complicated circuit if using discrete devices alone.

For this course we will be using a device called a PICAXE. These have been designed for the hobbyist and educational use, and are very easy to use.

For the exercises in this course we will be using a pre-build demonstration circuit board that uses the smallest PICAXE chip, connected to two inputs and 3 outputs.



Not all possible input and output functions of the PICAXE 08M2+ have been implemented on this board. It's configuration is:

Connector	LED	Pin	Port.Bit	Type	Functions
In 1		4	C.3	Pull-up	Switch, Digital in
In 2		3	C.4	Pull-up	Switch, Digital in, ADC
Out 1	Red	7	C.0	TTL	Digital out, DAC
Out 2	Amber	6	C.1	TTL	Digital out
Out 3	Green	5	C.2	Open Collector	Digital out, Tune

PICAXE PROJECT 1: Play a tune

In this first exercise we wil program the PICAXE to play a tune.

PREPARATION

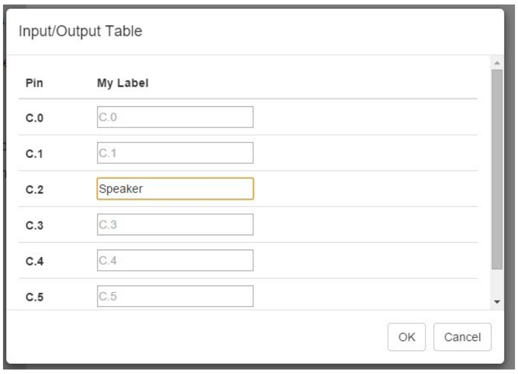
Connect a small loudspeaker to Out 3.

Using the Blockly language in PICAXE Editor 6, create a program that plays "Happy Birthday" when the circuit is powered on.

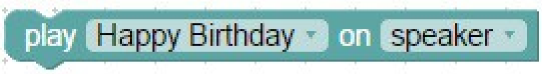
CODING

You will need to use these blocks:

Every program must start with a 'start' block. Right-click the block and select 'Input/Output Table'



This allows you to give meaningful names to the input and output ports. The loudspeaker is connected to port C.2, so give it your choice of name.

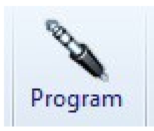


The 'play' block plays a pre-programmed tune on the output port you select.

You can select the tune you want to play.

PROGRAMMING THE UNIT

Connect the unit with the special USB lead. Click the 'Program' Icon:



Then immediately power on the circuit. The program will be downloaded to the microprocessor. When the download is finished the program will be run.

The download lead can now be disconnected.

The program will be run once each time the power is applied.

PICAXE PROJECT 2: Play a tune when a button is pressed

Our first program plays a tune once when the circuit is powered on, then it stops.

In this exercise we will enhance the program so that it plays the tune each time switch 1 is pressed.

CODING

The switch is on port C.3, so add a name for the switch to the Input/Output table.

You will need to add these new blocks to your program:

This block should come immediately after the 'start' block. It continually repeats the blocks inside it.

This block tests an input port to see if it is in the specified state, and if so, runs the blocks inside it. You need to select the input port for the switch.

Put this block inside the 'forever' block so that it runs repeatedly.

Your 'play' block goes inside this block.

PROGRAMMING THE UNIT

Program the circuit using the USB lead as you did for the previous exercise.

TESTING

When powered on, the program should do nothing.

Each time switch 1 is pressed it should play the tune.

PICAXE PROJECT 3: Play 2 tunes

Our previous program plays a tune when switch 1 is pressed. We will now enhance the program so that it plays a different tune if switch 2 is pressed.

CODING

Switch 2 is on C.4, so add a name for it to the Input/Output table.

You will not need any new type of block for this program, but you will need to repeat the 'if input' block, and the 'play' block within the 'forever' block.

For the 'if input' block select the second switch.

For the 'play' block, select a different tune (the output port is the same).

TESTING

When powered on, the program should do nothing.

Each time switch 1 is pressed it should play the first tune.

Each time switch 2 is pressed it should play the second tune.