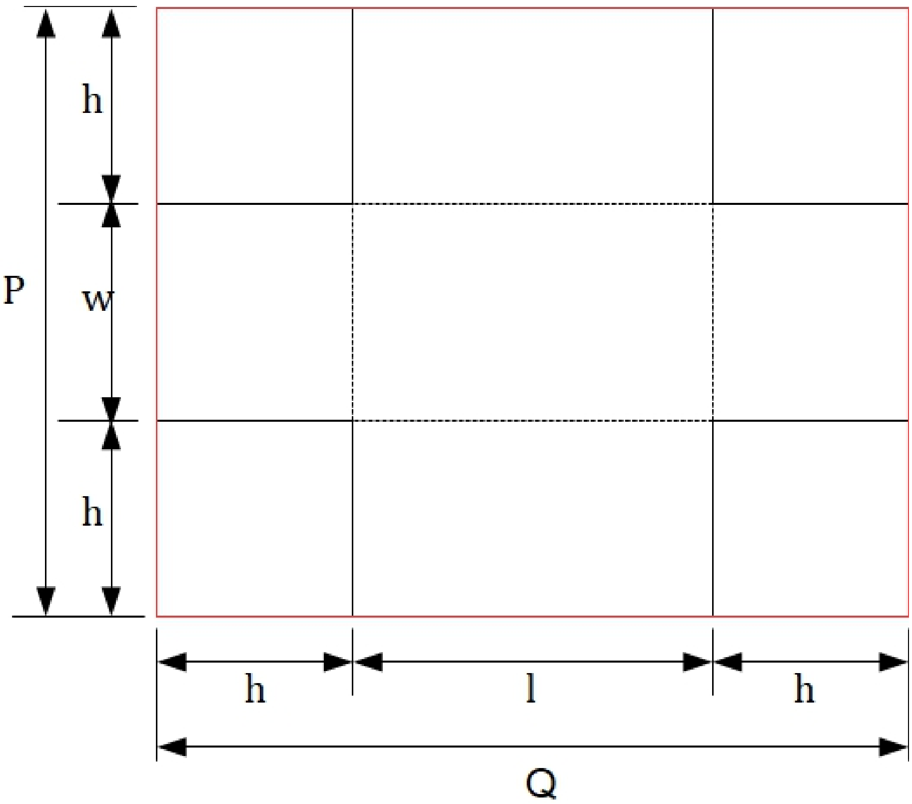
A PROJECT TO MAKE STORAGE BOXES

INTRODUCTION

It was desired to design a storage box that could be made easily, and in quantity, from re-cycled cardboard. The boxes were to take the form of a tray-like bottom section, with a loose-fitting lid. The boxes were to have the maximum volume that could be made from a sheet of cardboard of a given size. The card was to be cut and scored on a laser-cutter, that can handle sheets up to 20 x 30cm.

CALCULATIONS

The main task is to calculate the dimensions of a box with the maximum volume that can be cut from a sheet of card of a given size.



Consider a sheet of card of dimensions P by Q.

We wish to make a tray, or open-top box with the maximum possible internal volume, V.

Let the height, length and width of the box be h, l and w respectively.

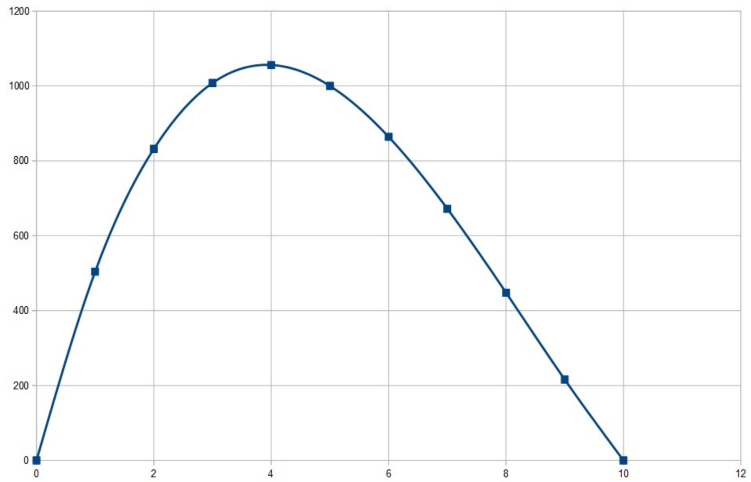
$$V = w \times l \times h$$

$$Q = l + 2h$$
$$l = Q - 2h$$

$$P = w + 2h$$
$$w = P - 2h$$

$$V = (P-2h)\times(Q-2h)\times h$$
$$V = (PQ-2Ph-2Qh+4h^2)\times h$$
$$V = (PQ-2(P+Q)h+4h^2)\times h$$
$$V = PQh-2(P+Q)h^2+4h^3$$

Plotting the volume for various values of height (in arbitrary units) gives a graph of the following form:



It can be seen that the maximum volume is achieved when the slope of the line is zero.

We note that the slope is found using the following differential expression:

$$\frac{d(aX^n)}{dy} = anX^{n-1}$$

(See Appendix for proof of the above)

Appling this rule to each term for the expression for volume, we obtain:

$$\frac{dV}{dh} = PQ-4(P+Q)h+12h^2$$
$$12h^2-4(P+Q)h+PQ=0$$

This is a a quadratic equation of the form:

$$ah^2+bh+c=0$$

Where

$$a=12,\ \ b=-4(P+Q),\ \ c=PQ$$

The solution is given by the well-known formula:

$$h=\frac{-b\mp\sqrt{b^2-4ac}}{2a}$$

The maximum capacity of the laser cutter is 200 x 300mm, but it is generally a good idea to leave a small margin (in this case 1 cm) around all edges, so we will set our dimensions thus: P = 18, Q=28.

$$b=-4(18+28)=-184$$
$$c=18\times 28=504$$

$$h = \frac{184 \mp \sqrt{(-184)^2 - (4 \times 12 \times 504)}}{2 \times 12}$$

$$h = \frac{184 \mp \sqrt{33856 - 24192}}{24}$$

$$h = \frac{184 \mp \sqrt{9664}}{24}$$

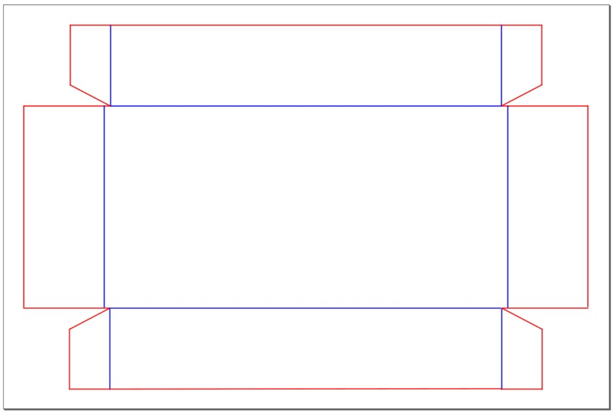
$$h = \frac{184 \mp 98.3}{24}$$

$$h = 11.76 \text{ or } 3.57$$

The value of h=11.76 is impossible, so we take 3.57 cm as the height of box sides that holds maximum volume.

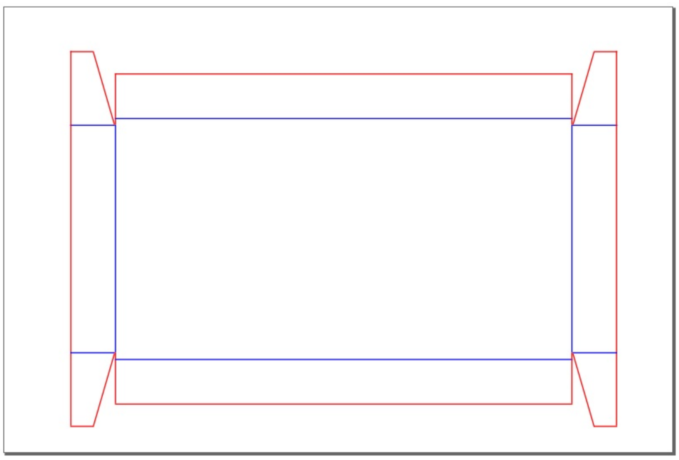
DESIGN

Using the dimensions calculated above, a drawing of the shape to be cut out was made using the InkScape 2D drawing program, thus:



The red lines are cutting lines, the blue lines are score lines for folding. The score lines on the end flaps are slightly offset to allow for the thickness of the cardboard (approx 2mm).

Having determined the dimensions of the box, it was then a simple matter to make a drawing for the lid. In this case the sides only needed to be 1cm deep:



THE FINISHED BOX

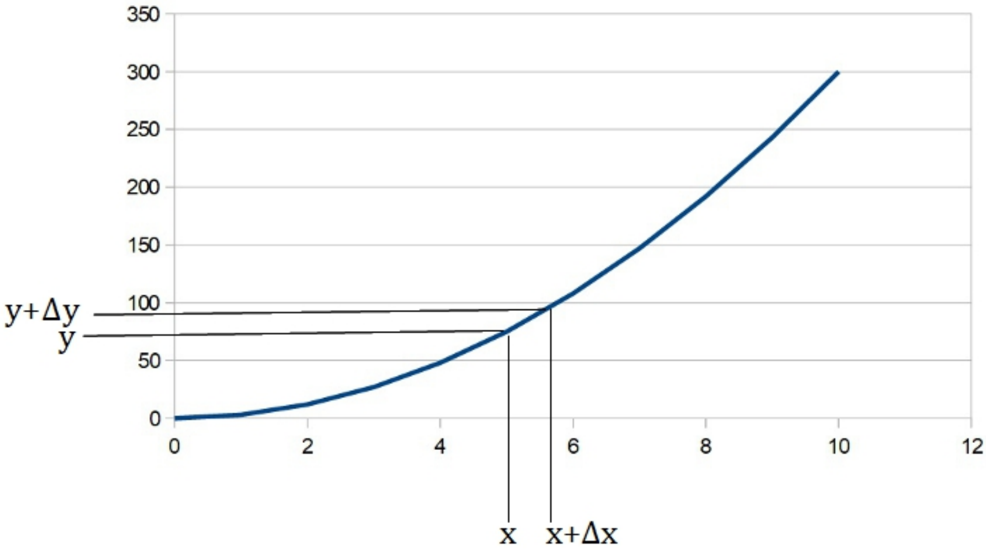
After scoring and cutting on the laser cutter, the card is folded and the tabs glued to make the finished box.

Having made one box, it is trivial to use the laser cutter to reproduce more boxes as needed.

APPENDIX

To show that the differential of an expression of the form: $y = ax^2$ is $\frac{dy}{dx} = 2\text{ax}$

The expression yields a graph of this form:



We wish to find the slope of the graph at a given value of (x,y). Consider a small increment Δx, giving the point (x+Δx, y+Δy). The slope of the straight line between these points is:

$$\frac{\Delta y}{\Delta x}$$

Express y in terms of x:

$$y = ax^2$$
$$y + \Delta y = a(x + \Delta x)^2 = ax^2 + 2\text{ax}\,\Delta x + \Delta x^2$$
$$\Delta y = ax^2 + 2\text{ax}\,\Delta x + \Delta x^2 - ax^2$$
$$\Delta y = 2\text{ax}\,\Delta x + \Delta x^2$$
$$\frac{\Delta y}{\Delta x} = 2\text{ax} + \frac{\Delta x^2}{\Delta x} = 2\text{ax} + \Delta x$$

To find the instantaneous slope at point x, we let Δx approach zero:

$$\frac{d\,y}{d\,x} = 2\text{ax}$$

Differentials of higher powers of x may be found using similar logic.