# FLASHING BOW-TIE

## INTRODUCTION

A bow-tie, with flashing lights was required for a theatrical costume. This document describes how it was constructed.

Although this is a simple circuit, it is an example of an important principle in electronics, the mathematics of which are included in this document, as the theory has very wide applicability.

*Illustration 1: Bow-tie and box of electronics*

## CIRCUIT

The circuit is based on an NE555 timer IC, connected as a basic square-wave oscillator.

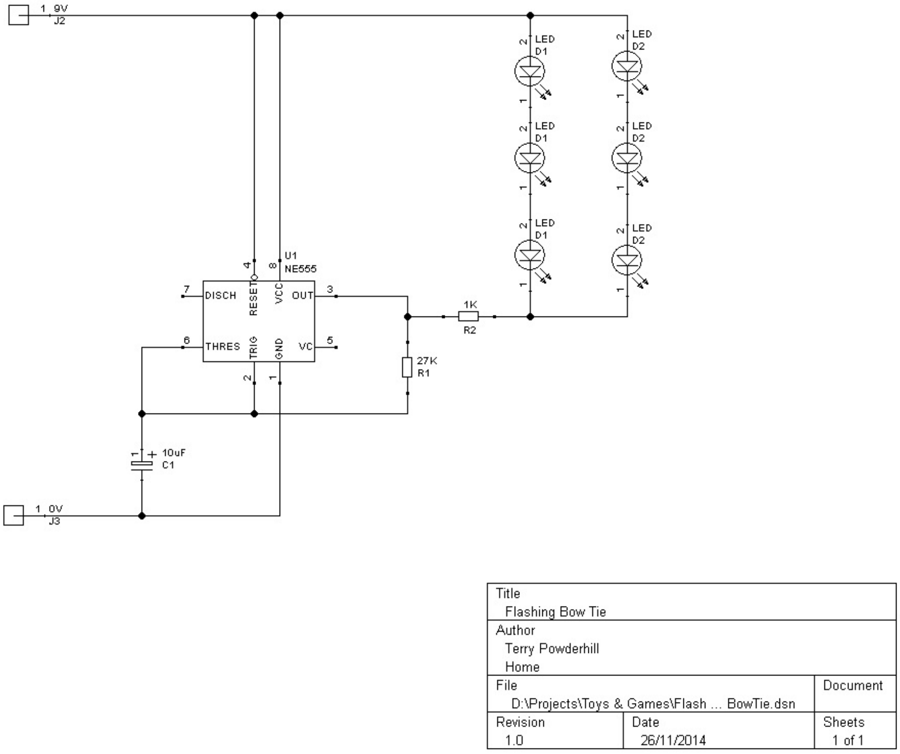


Illustration 2: Circuit Schematic

This circuit uses the 555 chip in an unusual way: It is more conventional to discharge the capacitor through the DISCHARGE pin, but this results in a pulsed output with a duty cycle less than 50%. Using the OUT pin to charge and discharge the capacitor provides a simple way to achieve a 50% duty cycle which was desired for this project.

When the output is high, capacitor C1 charges through R1. When the voltage on C1 reaches 2/3 of the supply voltage the THRESHOLD input switches the output off. Capacitor C1 then discharges through R1 until the voltage at TRIGGER drops to 1/3 of supply voltage, which causes the output to go high again and the sequence repeats.

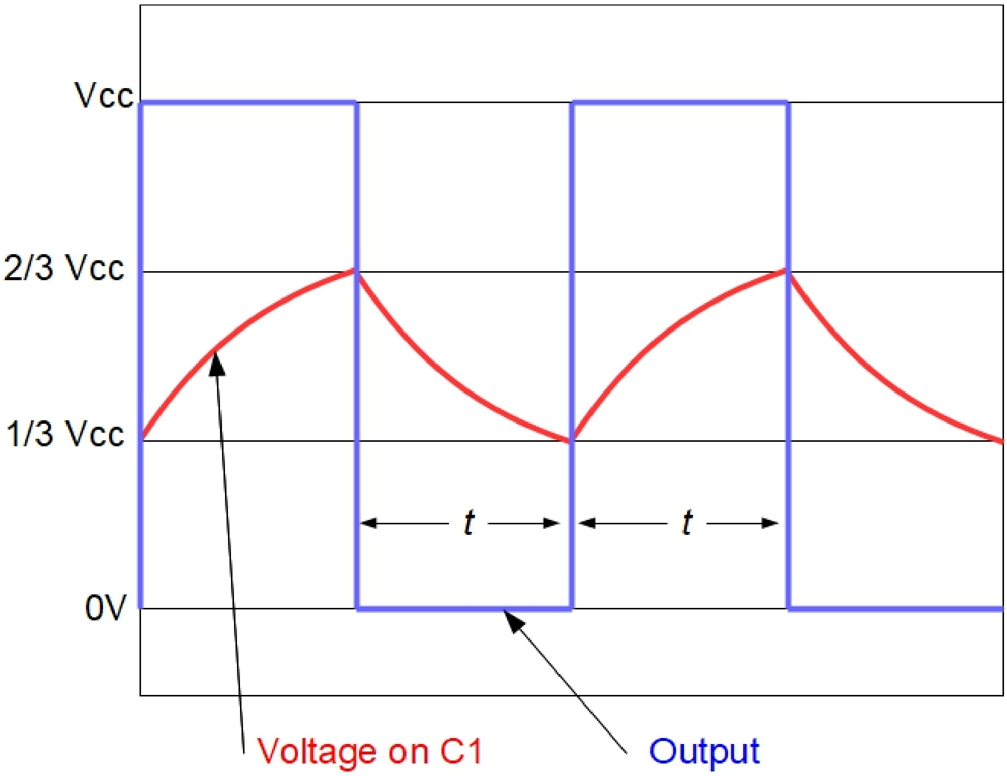


*Illustration 3: Waveforms*

When the output voltage is high, there is no voltage across the LEDs so they are off. When the output is low the supply voltage is present across the LEDs and and current is drawn through them via the current-limiting resistor R2.

## *Charging the Capacitor*

It can be shown (See Appendix) that the time to charge a capacitor C, through resistor R, from zero to V, with supply voltage $V_b$, is given by:

$$t = -\tau \ln\left(1 - \frac{V}{V_b}\right) \quad \text{Where} \quad \tau = R \times C$$

So, to charge the capacitor from 0 to 1/3 $V_b$ would take:

$$-\tau \ln\left(1 - \frac{1}{3}\right) = 0.405\,\tau$$

Similarly, to charge from 0 to 2/3 $V_b$ would take:

$$-\tau \ln\left(1 - \frac{2}{3}\right) = 1.0986\,\tau$$

So, to charge from 1/3 $V_b$ to 2/3 $V_b$ takes:

$$(1.0986-0.405)\tau=0.694\,\tau$$

With R = 27K and C = 10µF

$$t=0.694\times27\times10^{3}\times10\times10^{-6}=0.187\,seconds$$

The time to discharge from 2/3 Vs to 1/3 Vs is the same, so the frequency is:

$$\frac{1}{(2\times0.187)}=2.67\,Hz$$

Note that the frequency is independent of supply voltage.

## CIRCUIT – PHYSICAL

The circuit was built on a small piece of stripboard.

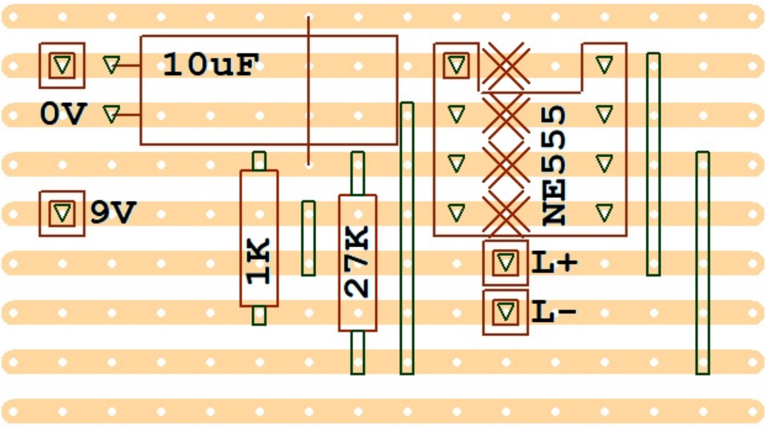


*Illustration 4: Stripboard Layout*

Note: In the finished circuit, the on/off switch was also mounted on the board, using spare tracks.

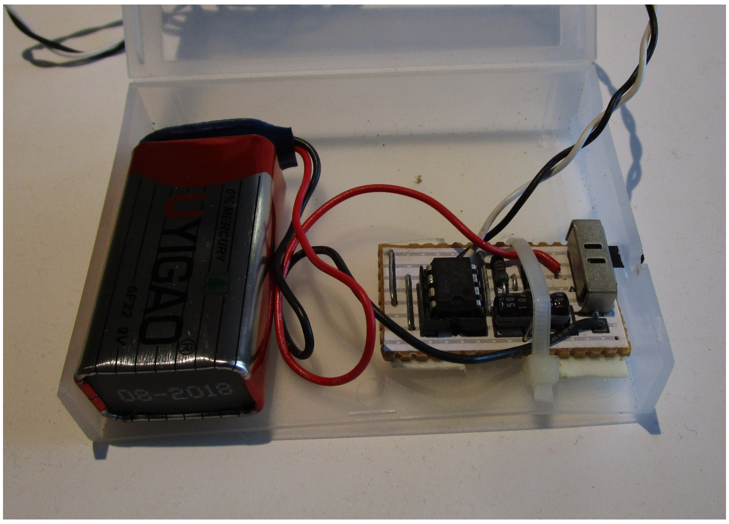
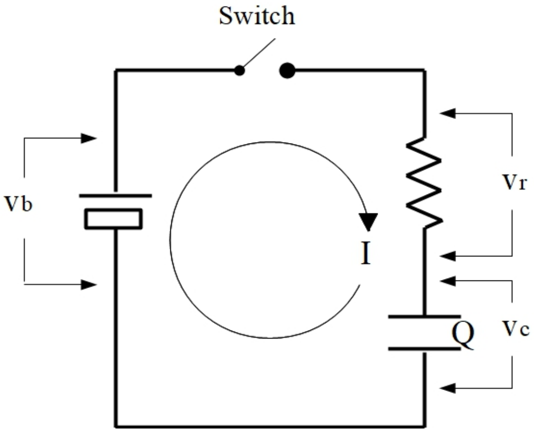


*Illustration 5: Enclosure with battery and stripboard.*

# Appendix – Charging a Capacitor Through A Resistor



We will adopt the usual convention that current flowing in the direction shown is the positive direction. Voltage across the battery (current flowing from negative to positive) will be considered to be positive, so the voltage drops across the resistor and capacitor (current flowing from positive to negative) is negative.

To analyse this circuit we use Kirchoff's second law (Voltage Rule): The sum of the voltage drops $\Delta V_i$, across any circuit elements that form a closed circuit is zero:

$$\sum_{i=1}^{i=n} \Delta V_i = 0$$

## Initial State

At t = 0, the switch is closed. The capacitor initially is uncharged and acts like a short circuit.

$$At\ t=0,\ Q=0,\ V_c=0,\ I=\frac{V_b}{R}$$

## Final State

After a very long time has passed the voltage difference between the capacitor plates equals the battery voltage. Since there is no current between points of equal voltage, the current will be zero.

$$As\ t\rightarrow\infty,\ Q\rightarrow CV_b,\ V_c\rightarrow V_b,\ I\rightarrow 0$$

## Voltage as a Function of Time

As the charge builds up on the capacitor plates, the current drops from the initial value, as derived above, to zero.

Using the voltage rule, and observing the convention for the sign of the voltages, , at time t we have:

$$V_b-V_r-V_c=0 \quad \therefore \quad V_b-I_{tR}-\frac{Q_t}{C}=0$$

Current is defined to be the flow of charge, by which we mean the rate of change of charge in time. So, given the conventions described above, the current is related to the charge on the capacitor by:

$$I=\frac{+dQ_t}{dt}$$

so

$$V_b - \frac{dQ_t}{dt}R - \frac{Q_t}{C} = 0 \quad \therefore \quad \frac{dQ_t}{dt}R = V_b - \frac{Q_t}{C} \quad \therefore \quad \frac{dQ_t}{dt} = \frac{1}{R}\left(V_b - \frac{Q_t}{C}\right)$$

This equation can be solved by the method of separation of variables. First put terms involving time (dt) on one side (the right):

$$dQ_t = \frac{1}{R}\left(V_b - \frac{Q_t}{C}\right)dt$$

Then put terms involving charge (dQ and Q) on the other side (the left).

$$\frac{dQ_t}{\left(V_b - \frac{Q_t}{C}\right)} = \frac{1}{R}dt$$

Re-arranging the terms on the LHS will facilitate integration:

$$\frac{CdQ_t}{CV_b - Q_t} = \frac{1}{R}dt \quad \therefore \quad \frac{dQ_t}{CV_b - Q_t} = \frac{1}{RC}dt \quad \therefore \quad \frac{dQ_t}{Q_t - CV_b} = -\frac{1}{RC}dt$$

The value RC is called the Time Constant of the circuit, and given the symbol τ. With V in Volts and R in Ohms, τ is in seconds.

$CV_b$ is the maximum value of Q as $t \to \infty$ , which we will call $Q_{max}$

Integrating both sides of the above equation we get:

$$\int_0^{Q_t} \frac{dQ_t}{Q_t - Q_{max}} = -\frac{1}{\tau}\int_0^t dt$$

For the LHS, this type of integral is solved thus: $\int \frac{c}{ax+b}dx = \frac{c}{a}\ln(|ax+b|)+K$

Since $Q_{max} > Q_t$ then $|Q_t - Q_{max}| = Q_{max} - Q_t$ . So we can evaluate the integrals thus:

$$\int_0^{Q_t} \frac{dQ_t}{Q_t - Q_{max}} = \ln(Q_{max} - Q_t) - \ln(Q_{max} - 0) = \ln\left(\frac{Q_{max} - Q_t}{Q_{max}}\right) = \ln\left(1 - \frac{Q_t}{Q_{max}}\right)$$

(From initial conditions we can see that the arbitrary constant K = 0)

For the RHS, this type of integral is solved thus: $\int dx = x$ so:

$$-\frac{1}{\tau}\int_0^t dt = -\frac{1}{\tau}(t-0) = -\frac{t}{\tau}$$

Thus:

$$\ln\left(1 - \frac{Q_t}{Q_{max}}\right) = -\frac{t}{\tau} \quad \therefore \quad t = -\tau\ln\left(1 - \frac{Q_t}{Q_{max}}\right)$$

Noting that $\frac{Q_t}{Q_{max}} = \frac{V_t}{V_b}$ we find that the time to charge the capacitor to a given voltage is:

$$t = -\tau\ln\left(1 - \frac{V_t}{V_b}\right)$$